Chapter-1

Matrices and Determinants

1. It is necessary for an adult to consume 86 grams of proteins, 58 grams of fats and 66 grams of carbohydrates. There are three types A, B, C of food available, which may be mixed to get desired food values. The following table gives the food analysis for the three types of foods:

Types of Food	**Food Value (grams)/100 grams**		
	Protein	**Fats**	**Carbohydrates**
A	**8**	**4**	**3**
B	**5**	**5**	**5**
C	**5**	**3**	**4**

What quantities of three types of foods will just be sufficient to provide necessary food values?

Solution: Let 100 gms. Be denoted by 1 unit. Let the quantities of three types of foods A, B and C be x, y and z units. Then according to the given information, we have

$$8x + 5y + 5z = 86$$

$$4x + 5y + 3z = 58$$

$$3x + 5y + 4z = 66$$

By Cramer's Rule (Determinant Method)

$$D = \begin{vmatrix} 8 & 5 & 5 \\ 4 & 5 & 3 \\ 3 & 5 & 4 \end{vmatrix} = 8\ (20 - 15) - 5\ (16 - 9) + 5\ (20 - 15)$$

$$= 8 \times 5 - 5 \times 7 + 5 \times 5$$

$$= 40 - 35 + 25 = 30 \neq 0$$

Since D $\neq$ 0, the system has a unique solution

$X = \frac{D_1}{D}$ $\quad$ $y = \frac{D_2}{D}$ $\quad$ $z = \frac{D_3}{D}$

$$D_1 = \begin{vmatrix} 86 & 5 & 5 \\ 58 & 5 & 3 \\ 66 & 5 & 4 \end{vmatrix} = 86\ (20 - 15) - 5\ (232 - 198) + 5\ (290 - 330)$$

$$= 86 \times 5 - 5 \times 34 + 5 \times (-40)$$

$$= 430 - 170 - 200$$

$$= 60$$

$$D_2 = \begin{vmatrix} 8 & 86 & 5 \\ 4 & 58 & 3 \\ 3 & 66 & 4 \end{vmatrix} = 8\ (232 - 198) - 86\ (16 - 9) + 5\ (264 - 174)$$

$$= 8 \times 34 - 86 \times 7 + 5 \times 90$$

$$= 272 - 602 + 450$$

$$= 120$$

$$D_3 = \begin{vmatrix} 8 & 5 & 86 \\ 4 & 5 & 58 \\ 3 & 5 & 66 \end{vmatrix} = 8\ (330 - 290) - 5\ (264 - 174) + 86\ (20 - 15)$$

$$= 8 \times 40 - 5 \times 90 + 86 \times 5$$

$$= 320 - 450 + 430 = 300$$

$X = \frac{D_1}{D} = \frac{60}{30} = 2$ $\quad$ $Y = \frac{D_2}{D} = \frac{120}{30} = 4$ $\quad$ $Z = \frac{D_3}{D} = \frac{300}{30} = 10$

Quantity of food A required = 2 units = 2 × 100

=200 grams

Quantity of food B required = 4 units = 4 × 100

= 400 grams

Quantity of food C required = 10 units = 10 × 100

= 1000 grams

2. A company produces three products every day. Their total production on a certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons, while the total production of first and third product is twice of the production of second product. Determine the production level of each product using Cramer's rule.

Solution: Let x, y, z be the production level of First, Second and Third product every day. Then according to the given information, we have

X + Y + Z = 45

Z – X = 8 or X + 0Y – Z = - 8

X – 2 Y + Z = 0

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} = 1\,(0 - 2)\ -1\,(1 + 1) + 1(-2 - 0)$$

= - 2 – 2 – 2 = - 6 $\neq$ 0

$X = \frac{D_1}{D}$ $\quad$ $Y = \frac{D_2}{D}$ $\quad$ $Z = \frac{D_3}{D}$

$$D_1 = \begin{vmatrix} 45 & 1 & 1 \\ -8 & 0 & -1 \\ 0 & -2 & 1 \end{vmatrix} = 45\,(0 - 2)\ -1\,(-8 - 0) + 1(16 - 0)$$

= 45 × (-2) -1 (-8) + × 16

= - 90 + 8 + 16

= - 66

$$D_2 = \begin{vmatrix} 1 & 45 & 1 \\ 1 & -8 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 1\,(-8 - 0) - 45\,(1 + 1) + 1(0 + 8)$$

= 1 × (-8) – 45 (2) + 1 × 8

= - 8 – 90 + 8 = - 90

$$D_3 = \begin{vmatrix} 1 & 1 & 45 \\ 1 & 0 & -8 \\ 1 & -2 & 0 \end{vmatrix} = 1\,(0 - 16) - 1\,(0 + 8) + 45\,(-2 - 0)$$

= 1 (-16) – 1 (8) + 45 × (-2)

= -16 – 8 – 90

= - 114

$X = \frac{D_1}{D} = \frac{-66}{-6} = 11$ $\qquad$ $Y = \frac{D_2}{D} = \frac{-90}{-6} = 15$

$Z = \frac{D_3}{D} = \frac{-114}{-6} = 19$

Production level of first product = 11 tons

Production level of Second product = 15 tons

Production level of Third product = 19 tons

3. A firm produces 3 products P_1, P_2 *and* P_3 requiring the mix-up of three materialsM_1, M_2 *and* M_3. The per unit requirement of each product for each material is given below:

$$\begin{array}{cc} & \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ A = & \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 1 \\ 1 & 4 & 2 \end{bmatrix} \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \end{array}$$

Using matrix method, find production of each product if firm has 850, 1250 and 650 units of the three materials respectively. Also find per unit cost of each product, if cost per unit of three materials are Rs 5, Rs 10 and Rs 12 Per unit respectively.

Solution: Let X, Y and Z be the given number of units which could be produced of the products P_1, P_2 *and* P_3 respectively.

According to the given data, we have

2 X + 3 Y + Z = 850

3 X + Y + 4 Z = 1250

X + Y + 2 Z = 650

The above system of equation can be expressed in the matrix from as

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 850 \\ 1250 \\ 650 \end{bmatrix}$$

A $\qquad$ **X** $\qquad$ **B**

The determinant of the coefficient matrix is:

$|A|$ = 2 (1 × 2 – 2 × 4) – 3 (3 × 2 – 1 × 4) +1 (3 × 1 – 1× 1)

= 2 (2 – 4) – 3 (6 – 4) + (3 – 1)

= 2 (-2) – 3 (2) + 1 (2)

= -4 – 6 + 2 = - 8 $\neq$ 0

Since $|A| \neq 0$ therefore A^{-1} exists. So first we find the A^{-1}.

$A^{-1} = \frac{1}{|A|}$adj. A, we first find out cofactors of matrix A.

C_{11} = (1 × 2 – 1 × 4) = 2 – 4 = -2

C_{12} = - (3 × 2 – 1 × 4) = - (6 – 4) = - 2

C_{13} = (3 × 1 – 1 × 1) = 3 – 1 = 2

C_{21} = - (3 × 2 – 1 × 1) = - (6 – 1)

C_{22} = (2 × 2 – 1 × 1) = (4 – 1) = 3

C_{23} = - (2 × 1 – 1 × 3) = - (2 – 3) = 1

C_{31} = (3 × 4 – 1 × 1) = (12 – 1) = 11

C_{32} = - (2 × 4 – 3 × 1) = - (8 – 3) = - 5

C_{33} = (2 × 1 – 3 × 3) = (2 – 9) = - 7

Cofactor matrix is $\begin{bmatrix} -2 & -2 & 2 \\ -5 & 3 & 1 \\ 11 & -5 & -7 \end{bmatrix}$

Now transpose of cofactor matrix is adj. A.

$$\text{Adj. A} = \begin{bmatrix} -2 & -5 & 11 \\ -2 & 3 & -5 \\ 2 & 1 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj. A} = \frac{1}{-8}\begin{bmatrix} -2 & -5 & 11 \\ -2 & 3 & -5 \\ 2 & 1 & -7 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{-8}\begin{bmatrix} -2 & -5 & 11 \\ -2 & 3 & -5 \\ 2 & 1 & -7 \end{bmatrix}\begin{bmatrix} 850 \\ 1250 \\ 650 \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} (-2 \times 850) + (-5 \times 1250) + (11 \times 650) \\ (-2 \times 850) + (3 \times 1250) + (-5 \times 650) \\ (2 \times 850) + (1 \times 1250) + (-7 \times 650) \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} -1700 - 6250 + 7150 \\ -1700 + 3750 - 3250 \\ 1700 + 1250 - 4550 \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} -800 \\ -1200 \\ -1600 \end{bmatrix} = \begin{bmatrix} 100 \\ 150 \\ 200 \end{bmatrix}$$

X = 100 units Y = 150 units Z = 200 units

Thus Production of product P_1 = 100 units

Thus Production of product P_2 = 150 units

Thus Production of product P_3 = 200 units

Second Part: Finding out per unit cost of each product.

The per unit cost of materials M_1, M_2 and M_3 can be represented by 3 × 1 matrix.

$$C = \begin{array}{c} \begin{bmatrix} 5 \\ 10 \\ 12 \end{bmatrix} \end{array}\begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array}$$

The per unit cost of production of each product

$$AC = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 1 \\ 1 & 4 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 5 + 3 \times 10 + 1 \times 12 \\ 3 \times 5 + 1 \times 10 + 1 \times 12 \\ 1 \times 5 + 4 \times 10 + 2 \times 12 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 30 + 12 \\ 15 + 10 + 12 \\ 5 + 40 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} 52 \\ 37 \\ 69 \end{bmatrix}$$

Thus Per unit cost of Product P_1 = Rs 52

Thus Per unit cost of Product P_2 = Rs 37

Thus Per unit cost of Product P_3 = Rs 69

4. Two television companies T V_1 and TV$_2$, both televise documentary programmes. T V_1 has two transmitting stations and TV$_2$ has three transmitting stations. All stations transmit different programmes. On an average the TV$_1$ station broadcasts 1 hour of documentary and 3 hours of variety programmes each day whereas each TV$_2$ station broadcasts 2 hours of documentary and $1\frac{1}{2}$ hours of variety programmes each day. The transmission of documentary and variety programmes costs approximately Rs 50 and Rs 200 per hour respectively. Express in matrix form and hence evaluate:

(i) The daily cost of transmission from each TV$_1$ and each TV$_2$ station.

(ii) The total number of hours which are devoted daily to documentary and to variety programmes by both companies.

(iii) The total daily cost of transmission incurred by both companies.

Solution: Let the hours of documentary and variety programmes be denoted by matrix A

$$A = \begin{array}{c} \\ TV_1 \\ TV_2 \end{array} \begin{array}{c} \begin{array}{cc} Doc. & Var. \end{array} \\ \begin{bmatrix} 1 & 2 \\ 2 & 3/2 \end{bmatrix} \end{array}$$

Let the transmission costs of documentary and variety programmes be denoted by matrix B.

$$B = \begin{bmatrix} 50 \\ 200 \end{bmatrix} \begin{array}{c} Doc. \\ Var. \end{array}$$

(i) The daily cost of transmission from each TV$_1$and TV$_2$ station is given by the matrix multiplication AB:

$$AB = \begin{array}{c} \\ TV_1 \\ TV_2 \end{array} \begin{array}{c} \begin{array}{cc} Doc. & Var. \end{array} \\ \begin{bmatrix} 1 & 2 \\ 2 & 3/2 \end{bmatrix} \end{array} \begin{bmatrix} 50 \\ 200 \end{bmatrix} \begin{array}{c} Doc. \\ Var. \end{array}$$

$$= \begin{array}{c} TV_1 \\ TV_2 \end{array} \begin{bmatrix} 650 \\ 400 \end{bmatrix}$$

Thus, the daily cost of transmission from TV_1 = Rs 650

And the daily cost of transmission from TV_2 = Rs 400

(ii) Let the number of transmitting stations TV$_1$ and TV$_2$ be denoted by matrix C:

$$C = \begin{array}{c} \begin{array}{cc} TV_1 & TV_2 \end{array} \\ [2 \quad 3] \end{array}$$

The total numbers of hours which are devoted daily to documentary and to variety programmes by both companies are given by the matrix multiplication CA:

$$CA = \begin{array}{c} \begin{array}{cc} TV_1 & TV_2 \end{array} \\ [2 \quad 3] \end{array} \begin{array}{c} \begin{array}{cc} Doc. & Var. \end{array} \\ \begin{bmatrix} 1 & 3 \\ 2 & 3/2 \end{bmatrix} \end{array} \begin{array}{c} TV_1 \\ TV_2 \end{array} = \begin{array}{c} \begin{array}{cc} Doc. & Var. \end{array} \\ [8 \quad 10.5] \end{array}$$

Thus 8 hours and 10.5 hours respectively are devoted daily to documentary and variety programmes by both the companies.

(iii) The total daily cost of transmission incurred by both campanies is given C. AB:

$$C.AB = [2 \quad 3] \begin{bmatrix} 650 \\ 400 \end{bmatrix} = [2500]$$

Thus the total daily cost of transmission incurred by both companies is Rs 2500.

5. Show that the matrix A = $\begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ satisfies the equation A^3 - $7A^2$ - 5A + 13I = 0

Solution: $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

$$5A = 5 \begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 25 & 15 & 5 \\ 10 & -5 & 10 \\ 20 & 5 & 15 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 13 & 14 \\ 16 & 9 & 6 \\ 34 & 14 & 15 \end{bmatrix}$$

$$7A^2 = 7 \begin{bmatrix} 35 & 13 & 14 \\ 16 & 9 & 6 \\ 34 & 14 & 15 \end{bmatrix} = \begin{bmatrix} 245 & 91 & 98 \\ 112 & 63 & 42 \\ 238 & 98 & 105 \end{bmatrix}$$

$A^3 = A^2 \times A$

$$A^3 = \begin{bmatrix} 35 & 13 & 14 \\ 16 & 9 & 6 \\ 34 & 14 & 15 \end{bmatrix} \times \begin{bmatrix} 5 & 3 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 257 & 106 & 103 \\ 122 & 45 & 52 \\ 258 & 103 & 107 \end{bmatrix}$$

$$13I = 13\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

$A^3 - 7A^2 - 5A + 13I$

$$= \begin{bmatrix} 257 & 106 & 103 \\ 122 & 45 & 52 \\ 258 & 103 & 107 \end{bmatrix} - \begin{bmatrix} 245 & 91 & 98 \\ 112 & 63 & 42 \\ 238 & 98 & 105 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & -5 & 10 \\ 20 & 5 & 15 \end{bmatrix} + \begin{bmatrix} 13 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

6.The Dummy Electronics produces 3 types of radio sets Type 1, Type 2 and Type 3. Type 1 contains 1 transistor, 10 resistors and 5 capacitors, while Type 2 contains 2 transistor, 18 resistors and 7 capacitors and type 3 contains 3 transistors, 24 resistors and 10 capacitors. Monthly demand for Type 1, Type 2 and Type 3 radio sets is 100 units, 250 units and 80 units respectively. Arrange this information in matrix form for determination of monthly consumption of transistors, resistors and capacitors. Solution of system of equations is not required.

Solution:

$$X = \begin{array}{r|ccc|} & Audio1 & Audio\ 2 & Audio\ 3 \\ Transistors & 1 & 2 & 3 \\ Resistors & 10 & 18 & 24 \\ Capacitors & 5 & 7 & 10 \end{array}$$

The weekly demand is represented by a column matrix Y.

$$Y = \begin{array}{r|c|} Audio\ 1 & 100 \\ Audio\ 2 & 250 \\ Audio\ 3 & 80 \end{array}$$

The weekly consumption of transistors, resistors and capacitors is given the matrix product XY

$$XY = \begin{array}{r|ccc|} & Audio\ 1 & Audio\ 2 & Audio\ 3 \\ Transistor & 1 & 2 & 3 \\ Resistors & 10 & 18 & 24 \\ Capacitors & 5 & 7 & 10 \end{array} \begin{bmatrix} 100 \\ 250 \\ 80 \end{bmatrix}$$