

Mathematics of Finance

1. Find the amount of Rs 2000 after two years at 12% per annum, if compounding is done:

- (1) Annually (2) Semi-annually
(3) Quarterly (4) Monthly

Solution: $A = P \left(1 + \frac{r}{m}\right)^{m \times t}$

Where A = Amount

r = Rate of interest

t = time in years

m = Number of times interest is compounded in a year.

Given: P = Rs 2000 r% = 12%

t = 2 year

(1) When the interest is compounded annually, m = 1.

$$A = 2000 \left(1 + \frac{0.12}{1}\right)^{1 \times 2} = 2000 (1 + 0.12)^2$$

$$= 2000 (1.12)^2 = \text{Rs } 2508.80$$

(2) When the interest is compounded Semi annually, m = 2.

$$A = 2000 \left(1 + \frac{0.12}{2}\right)^{2 \times 2} = 2000 (1 + 0.06)^2$$

$$= 2000 (1.06)^2 = \text{Rs } 2524.95$$

(3) When the interest is compounded Quarterly, m = 4.

$$A = 2000 \left(1 + \frac{0.12}{4}\right)^{4 \times 2} = 2000 (1 + 0.03)^8$$

$$= 2000 (1.03)^8 = \text{Rs } 2533.54$$

(3) When the interest is compounded monthly, m = 12.

$$A = 2000 \left(1 + \frac{0.12}{12}\right)^{12 \times 2} = 2000 (1 + 0.01)^{24}$$

$$= 2000 (1.01)^{24} = \text{Rs } 2539.47$$

2. Which is better investment, 12% per year compounded monthly or 12.5% per year compounded semiannually?

Solution: Option 1: 12% per year compounded monthly

$$M = 12, \quad r = 12\% = 0.12$$

$$\begin{aligned} \text{Effective rate of interest } (r_e) &= \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.12}{12}\right)^{12} - 1 \\ &= (1 + 0.01)^{12} - 1 \\ &= (1.01)^{12} - 1 \\ &= 1.1268 - 1 \\ &= 0.1268 = 12.68\% \end{aligned}$$

Option 2: 12.5% per year compounded semiannually

$$M = 2 \quad r = 12.5\% = 0.125$$

$$\begin{aligned} \text{Effective rate of interest } (r_e) &= \left(1 + \frac{r}{m}\right)^m - 1 \\ &= \left(1 + \frac{0.125}{2}\right)^2 - 1 \\ &= (1 + 0.0625)^2 - 1 \\ &= (1.0625)^2 - 1 \\ &= 1.1289 - 1 \\ &= 0.1289 = 12.89\% \end{aligned}$$

Since r_e under option 2 is higher than r_e under option 1, Option 2 is better investment at 12.5% per year compounded Semi – annually is better.

3. Find the present value of Rs 2000 due at end of 3 years is money is worth 6% Compounded semi – annually.

Solution: Amount (A) = Rs 2000

$$\text{Rate of interest } (r) = 6\% = 0.06$$

$$\text{Time } (t) = 3 \text{ years}$$

$$m = 2$$

$$A = P \left(1 + \frac{r}{m} \right)^{m \times t}$$

$$2000 = P \left(1 + \frac{0.06}{2} \right)^{2 \times 3}$$

$$2000 = P (1 + 0.03)^6$$

$$2000 = P (1.03)^6$$

$$P = \frac{2000}{(1.03)^6}$$

$$P = \frac{2000}{1.1941} = \text{Rs } 1674.90$$

Therefore, Present Value of Rs 2000 due at the end of 3 years if money is worth 6% compounded Semi – annually is Rs 1674.90.

4. A money lender charges ‘interest’ at the rate of 3 rupees per Rs100 rupees per month payable in advance. What effective rate does he charge per annum?

Solution: Since the money lender charges interest at the rate of Rs 3 per Rs 100 per month, payable in advance therefore Rs 3 may be treated as interest on Rs 97 for one month. Consequently, interest rate per month is $\frac{r}{m}$ $\frac{3}{97}$. Thus, the equivalent rate r_e per annum is given by

$$\begin{aligned} (r_e) &= \left(1 + \frac{r}{m} \right)^m - 1 \\ &= \left(1 + \frac{3}{97} \right)^{12} - 1 \\ &= \left(\frac{100}{97} \right)^{12} - 1 \\ &= (1.031)^{12} - 1 \\ &= 1.4413 - 1 \\ &= 0.4413 = 44.13\% \end{aligned}$$

5. Sum of money at 4% p.a. compounded annually gives an interest of Rs 25 in second years. Find the interest of third year.

Solution: Let P the original principal which being invested at 4% compounded annually.

The amount at the end of first year = P (1.04)

The amount at the end of second year = P (1.04)²

The amount at the end of third year = P (1.04)³

Interest for the second year = P (1.04)² - P (1.04)

But interest for the second year = 25

$$P (1.04)^2 - P (1.04) = 25$$

$$P (1.04) (1.04 - 1) = 25 \dots\dots\dots (1)$$

$$\begin{aligned} \text{The interest for the third year} &= P (1.04)^3 - P (1.04)^2 \\ &= P (1.04)^2 (1.04 - 1) \\ &= P (1.04) (1.04) (0.04) \\ &= 25 \times 1.04 \text{ (using eq. 1)} \\ &= \text{Rs } 26 \end{aligned}$$